

Halstead Community Primary School

Written and Manipulatives Policy Part A Addition & Subtraction



**Approved by the Full Governing Body
Signed: Patricia Dunmall**

**Date:
Review date: November 2017**

Written and Manipulatives Policy Part A

Introduction

This policy contains the key pencil and paper procedures that are to be taught throughout the school. It has been written to ensure consistency and progression throughout the school. Although the focus of this policy is on pencil and paper procedures and the use of manipulatives it is important to recognise that the ability to calculate mentally lies at the heart of numeracy therefore please refer to the [Mental Calculation Policy](#). Part A of the policy outlines the written and manipulative methods for addition and subtraction and Part B outlines these for multiplication and division.

Aims

- To ensure a consistent and progressive approach exists within the school to secure good progress in written calculations and use of manipulative.
- To ensure that mental calculation is not at the exclusion of written recording and use of physical apparatus. Representative strategies and mental calculation are complementary strategies as in all methods there is an element of mental processing.
- For children to be able to use written recordings and manipulatives to clarify their thinking and support/extend the development of more fluent and sophisticated mental strategies.
- For children to select and utilise methods of written calculation and manipulatives independently and be given the freedom and encouragement to develop their own methods. Although each method will be taught in the year group specified, children should not be discouraged from using previously taught methods with which they are secure, while the new concepts are becoming embedded. In addition if children are secure in one form of calculation differentiation should provide children the opportunity to progress to a more sophisticated form.
- For children to reflect upon which method to use to solve a problem and ask questions such as 'Can I do this in my head?' 'Can I do this in my head or do I need equipment to help me?', 'Do I need to use a written method?' then 'Is my answer sensible?'
- For children to be able to clearly explain methods of recording/representation and justify why their answers are correct using sound mathematical vocabulary. Therefore strong speaking and listening opportunities underpin good mathematics teaching.
- For KS2 children to develop an efficient, reliable, compact written method of calculation for each operation that they can apply with confidence and understanding when undertaking calculations that they cannot carry out solely mentally.
- To share progress in written calculations with parents so that they have the confidence and knowledge to support their children at home with their mathematical development.

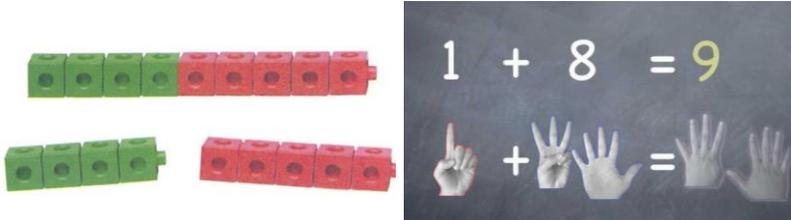
The objectives and content for this policy was originally provided by Chartwell Primary Maths September 2013

Addition

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition, which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers.

Mental Strategies required to add successfully

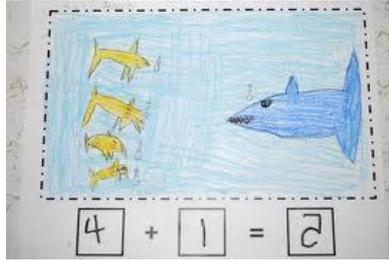
- Count forward in 1's, 2's, 5's and 10's etc...
- Recall all addition pairs to $9 + 9$ and complements in 10, (such as $7 + 3 = 10$) and apply them in calculations
- Add mentally a series of one-digit numbers, (such as $5 + 8 + 4$);
- Add multiples of 10 (such as $60 + 70$) or of 100, (such as $600 + 700$) using the related addition fact, $6 + 7$, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways. It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for addition

<p><u>Counting in ones</u></p> <p>Starting from 0 and then from any number</p>	<p>Counting out loud and practicing 1:1 correspondence (knowing that each object is a separate unit) It is also important that each number represents a group of objects (e.g. $3 = 3$ teddies)</p> 
<p><u>Practical Addition (first 'count all' and then 'count on')</u></p> <p>Count all: $2 + 3$ is counted 1, 2 and then 3, 4, 5 (out loud)</p> <p>Count on: $2 + 3$ is counted 2 and then 3, 4, 5</p>	

Simple addition using picture jottings

Drawing a picture

There were 4 yellow sharks and 1 blue. How many sharks were there altogether?



$$3 + 4 = 7$$

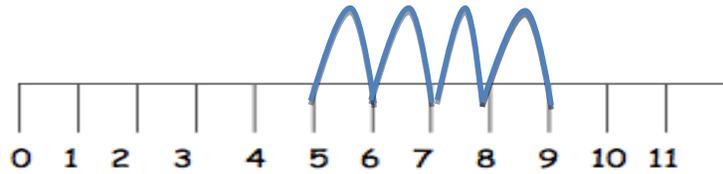


Dots or tally marks

3 kids were on a bus and then 4 more got on. How many were on the bus in total?

Counting in ones along a number line/track

5 children are at school. 4 children arrive late. How many children are at school now?



Children could use a pre-drawn number line and then begin to create their own.

$$5 + 4 = 9$$

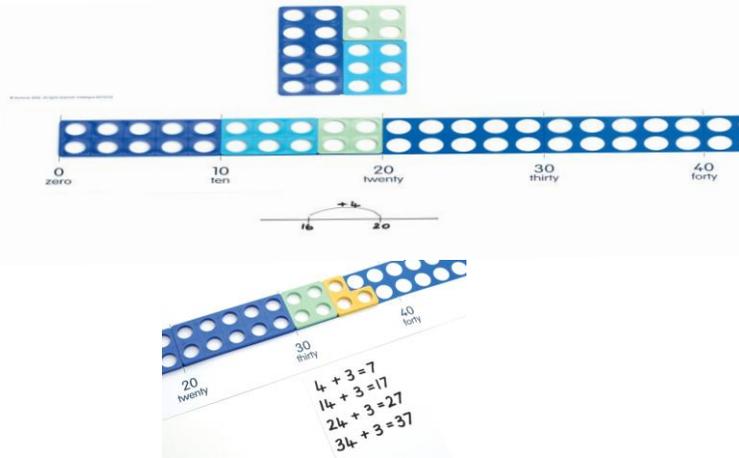
Practical Addition

Use of manipulatives

To help children to learn number facts and visualize quantities and what digits represent.

To look for patterns and relationships in number.

Use of Numicon number line to add amounts.



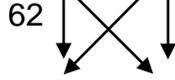
Practical and informal partitioning

Use of practical apparatus such as Numicon and multibase to partition and present place value of digits.

Place value cards can be used to additional support understanding of place value.

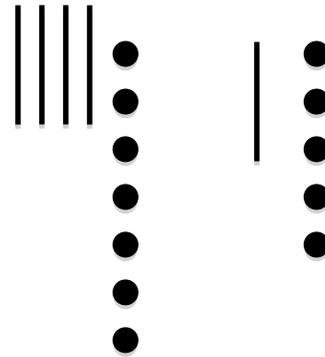


e.g. $47 + 12 =$



$50 + 9 = 59$

or $47 + 12 =$

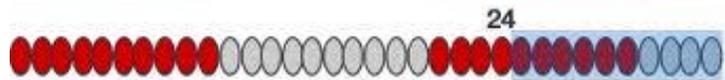
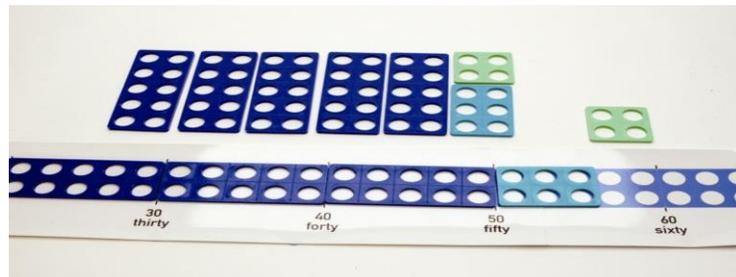


Using Addition facts to 10 to bridge the ten during addition.

Here Numicon is used to bridge the ten. E.g. in $56 + 8$ the 8 is split into two 4's in order to form a number bond of $6 + 4$. This enables the next multiple of 10 (60) to be reached before adding on the remaining 4.

Bead lines can also be used to show this method.

$24 + 10 = 24 + 6 + 4$



The empty number line

The mental methods that lead to column addition generally involve partitioning. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.

The empty number line helps to record the steps on the way to calculating the total.

One step in their develop when using a number line is to first be able to on in tens and of course ones.

$$8 + 7 = 15$$



$$48 + 36 = 84$$



or



Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10

Partitioning by horizontal expansion method

The next stage is to record mental methods using partitioning into tens and ones separately. Add the tens and then the ones to form partial sums and then add these partial sums.

Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This method builds on mental methods as each part is calculated mentally and recorded. It also makes the value of digits clear to children.

Before calculation children should be able to make a

Record steps in addition using partitioning:

$$47 + 76$$

$$47 + 70 = 117$$

$$117 + 6 = 123$$

or

$$47 + 76$$

$$40 + 70 = 110$$

$$7 + 6 = 13$$

$$110 + 13 = 123$$

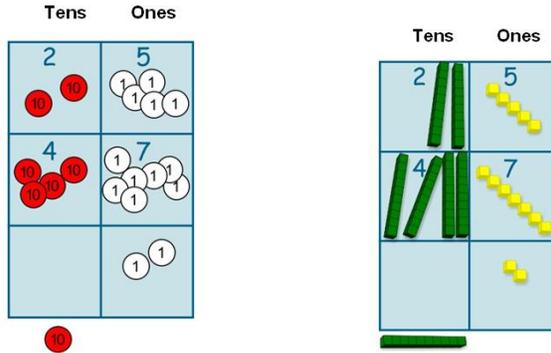
Partitioned numbers are then written under one another, for example :

47	40	7		
76	= 70	6		
	110	13	123	

sensible estimate (e.g. $487+546=$ is approximately $500+500=1000$) so they can check the reasonableness of their answer.

Vertical expansion method

Vertical expansion using multibase and place value counters.



Vertical expansion method

Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums initially the tens, not the ones, are added first, following mental methods. The total of the partial sums can be found by adding them together.

The addition of the tens in the calculation $47 + 76$ is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. As children gain confidence, ask them to start by adding the ones digits first every time.

Write the numbers in columns.

Adding the tens first:

$$\begin{array}{r} 47 \\ + 76 \\ \hline 110 \text{ (40+70)} \\ \underline{13} \text{ (7+6)} \\ 123 \end{array}$$

Adding the ones first:

$$\begin{array}{r} 47 \\ + 76 \\ \hline 13 \text{ (7+6)} \\ \underline{110} \text{ (40+70)} \\ 123 \end{array}$$

Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to adding the ones digits first consistently.

The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practicing the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Ensure that digits are kept in the correct columns throughout and columns may be labeled with H, T and U to show place value.

Compact column method

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

$$\begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ 11 \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

$$\begin{array}{r} 0.25 \\ 1.863 \\ + 1.2 \\ \hline 3.313 \\ \hline 1.1 \end{array}$$

Subtraction

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction, which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers. These will be introduced to children after they have understood the term subtraction by using objects and pictures to reinforce the idea. Children will always have had experience of using a numbered number line as well.

Mental Strategies required to subtract successfully

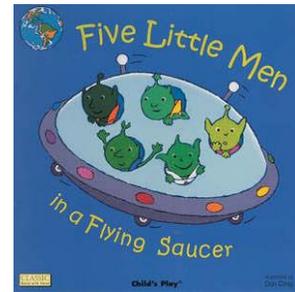
- Count back in ones, and tens from any number
- Recall all addition and subtraction facts to 10 and 20;
- Subtract multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

It is important that children’s mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for subtraction.

Counting backwards in ones

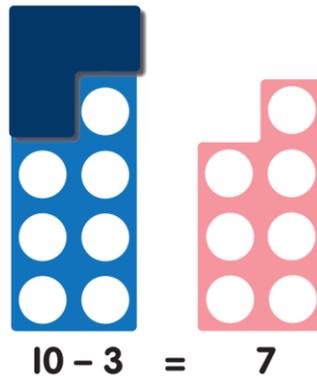
Starting from 10 and then from any number

Counting out loud and singing number rhymes e.g. 'Five current buns' or 'ten green bottles', using visual pictures and puppets to show the process of getting less



Practical subtraction (first 'count all' then 'count on')

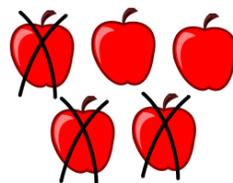
Practical 1:1 correspondence of finding the first number and taking away the second to find out what is left.
I found 3 pebbles on a beach but I lost one! How many did I have left?



Simple subtraction using picture jottings

Drawing a picture

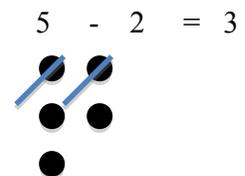
I had 5 apples but my teacher ate 3 of them. How many did I have left?



$5 - 3 = 2$



$5 - 2 = 3$

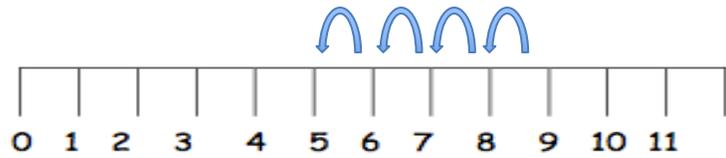


Dots or tally marks

There were 5 people of the bus but 2 got off at the first stop. How many people were still on the bus?

Counting back in ones along a number line/track

9 children are at school. 4 children go home because they feel sick. How many children are left in school? Children could use a pre-drawn number line and then begin to create their own.



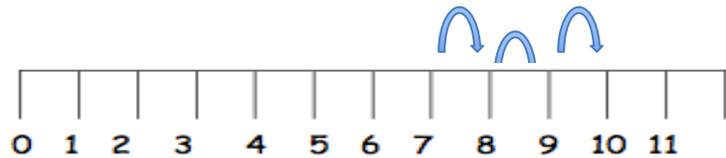
$$9 - 4 = 5$$

Children should use counting back briefly in their development when subtracting small single digit numbers and then move to using counting on in terms of finding the difference predominantly.

Finding the difference

There are 10 children in our class today and 7 of them are having school dinner. How many are having packed lunch.

Children are taught to count on from the smallest number to find the difference. (This can be done in their head or on a number line)



Using Addition facts to 10 to bridge the ten during subtraction by counting up to find the difference.

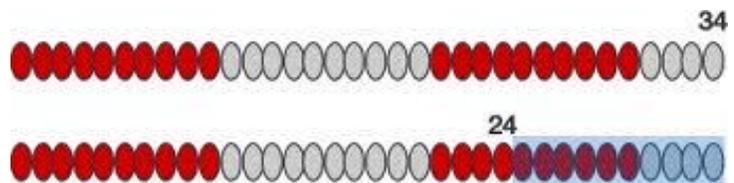
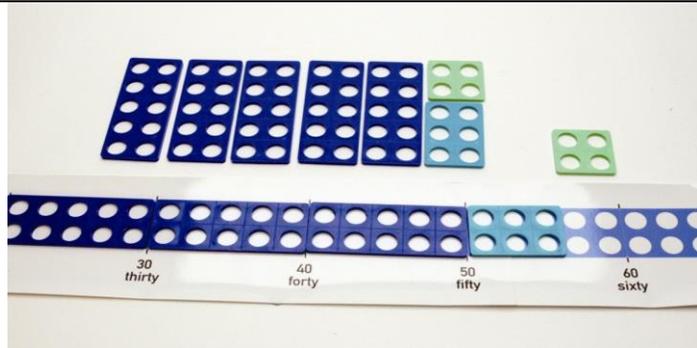
Here the use of a Numicon can be used to help bridge the ten when counting up to find the difference.

E.g. in $64 - 56 =$

4 in added to the 56 to reach 60 and then 4 again to reach 64. Thereby finding the difference of 8 by counting up.

Bead lines can also be used to show this method.

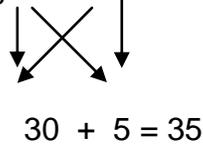
E.g. $34 - 24 = 6 + 4$



Practical and informal partitioning

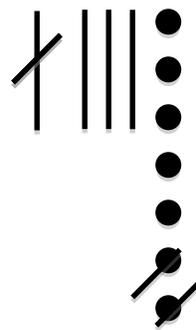
Use of practical apparatus such as multibase and Numicon.

e.g. $47 - 12 =$



This works when the units do not bridge the ten.

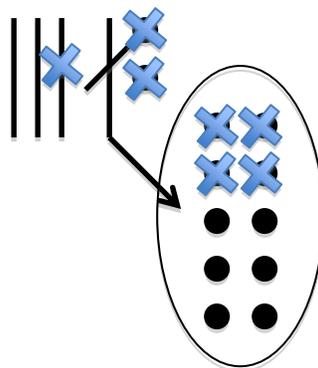
or $47 - 12 = 35$



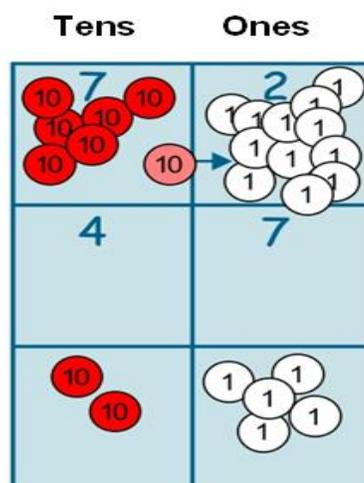
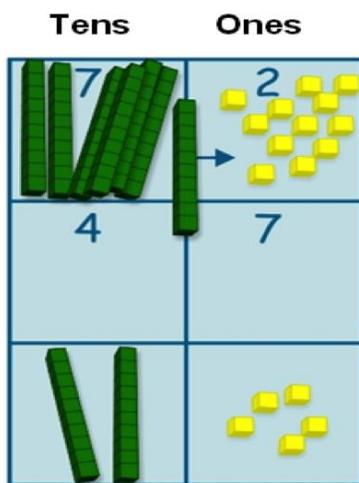
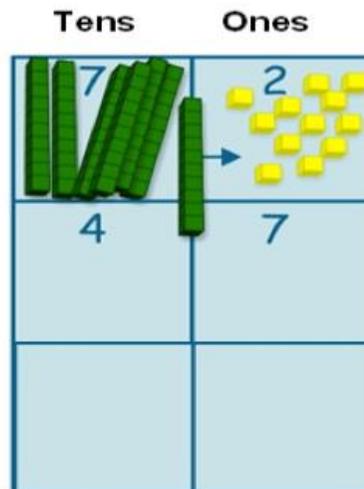
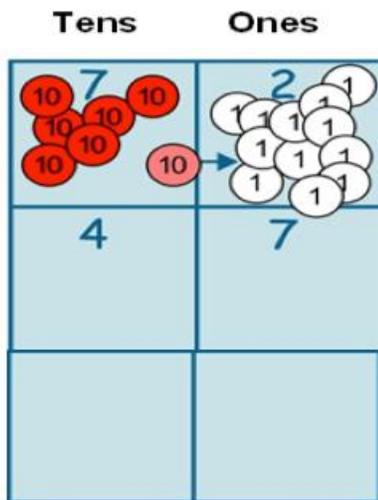
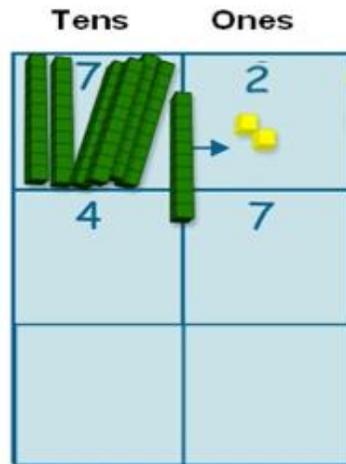
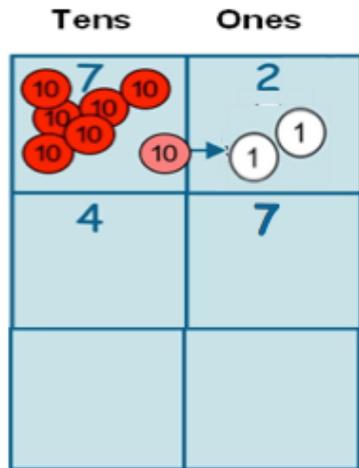
Practical partitioning where the units bridge the ten/hundreds

As there are only 2 units children should be taught to **exchange** 1 ten for ten units. This now means that there are 12 units and 2 tens (still 42). Children are then able to subtract 6 units and 1 ten, as in the previous example.

$42 - 16 = 26$



Example: $72 - 47$, **adjustment from the tens to the units** so that units can be taken away using multibase and place value counters.

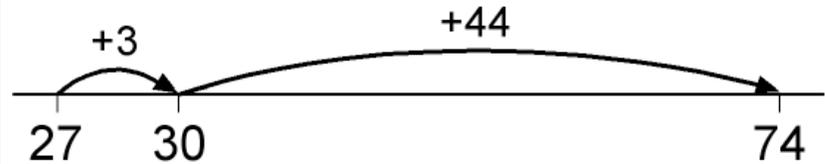
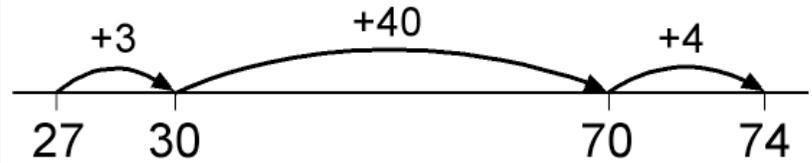


Using an empty number line to find the difference by counting up

The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeepers method).

With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as $57 - 12$, $86 - 77$ or $43 - 28$.

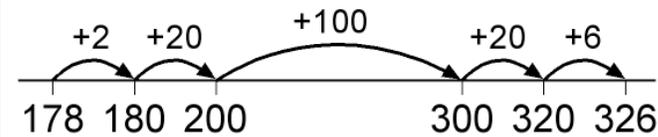
$74 - 27 =$
or:



With three-digit numbers the number of steps can again be reduced, enabling children to work out answers to calculations such as $326 - 178$ first in small steps and then more compact by using knowledge of complements to 100

The most compact form of recording becomes reasonably efficient.

$326 - 178 =$



$100 + 40 + 8 = 148$

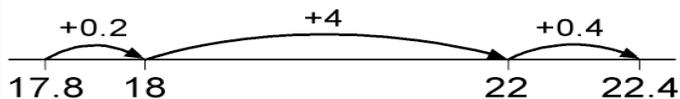


$126 + 22 = 148$

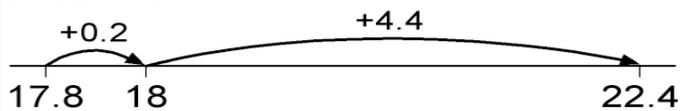
The method can successfully be used with decimal numbers.

This method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of KS2 is towards the

$22.4 - 17.8 =$



$4 + 0.6 = 4.6$



$4.4 + 0.2 = 4.6$

lower end of level 4 or below.	
<p><u>Partitioning</u></p> <p>Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For $74 - 27$ this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 7 in turn.</p> <p>This use of partitioning is a useful step towards the most commonly used column method, decomposition</p>	<p>Subtraction can be recorded using partitioning:</p> $74 - 27$ $74 - 20 = 54$ $54 - 7 = 47$
<p><u>Expanded layout, leading to column method (Decomposition)</u></p> <p>Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.</p> <p>This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.</p> <p>The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practicing the expanded method will depend on how secure the children are in their recall of number facts</p>	<p>Example: $563 - 241$, no adjustment or decomposition needed</p> <p>Expanded method</p> $\begin{array}{r} 500 \ 60 \ 3 \\ - \ 200 \ 40 \ 1 \\ \hline 300 \ 20 \ 2 \end{array}$ <p>Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.</p>

and with partitioning.

Example: $563 - 317$, **adjustment from the tens to the units** so that units can be taken away

$$\begin{array}{r} 50 \\ 500 + \cancel{60} + \cancel{3} \\ - \underline{200 + 40 + 6} \\ \hline \underline{300 + 10 + 7} = 317 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, how there is a "snag" with the units and the need to exchange a ten. To release units $60 + 3$ can be partitioned into $50 + 13$. The subtraction of the tens becomes '13 minus 6,

This method can be quite labour intensive which is why manipulative strategies are preferable to demonstrate this process which can then lead straight to the method.

Compact Method

$$\begin{array}{r} 5 \\ 5 \cancel{6} 3 \\ \underline{2 \ 4 \ 6} \\ \hline \underline{3 \ 1 \ 7} \end{array}$$

Ensure that children can explain the compact method, referring to the real value of the digits. They need to understand that they are repartitioning the $60 + 3$ as $50 + 13$.

Example: $563 - 271$, **Compact Method**

$$\begin{array}{r} 4 \\ \cancel{5} 6 3 \\ \underline{2 \ 7 \ 1} \\ \hline \underline{2 \ 9 \ 2} \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how $500 + 60$ can be partitioned into $400 + 160$. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten. **Ensure that children are confident to explain how the numbers are repartitioned and why**

