Halstead Community Primary School

Written and Manipulatives Policy
Part B
(Multiplication and Division)

Approved by the Full Governing Body
Signed: Patricia Dunmall (Chair of Governors)

Date:
Review Date: November 2017

Written and Manipulatives Policy Part B

Introduction

This policy contains the key pencil and paper procedures that are to be taught throughout the school. It has been written to ensure consistency and progression throughout the school. Although the focus of this policy is on pencil and paper procedures and the use of manipulatives it is important to recognise that the ability to calculate mentally lies at the heart of numeracy therefore please refer to the Mental Calculation Policy. Part A of the policy outlines the written and manipulative methods for addition and subtraction and Part B outlines these for multiplication and division.

Aims

- To ensure a consistent and progressive approach exists within the school to secure good progress in written calculations and use of manipulative.
- To ensure that mental calculation is not at the exclusion of written recording and use of physical apparatus. Representative strategies and mental calculation are complementary strategies as in all methods there is an element of mental processing.
- For children to be able to use written recordings and manipulatives to clarify their thinking and support/extend the development of more fluent and sophisticated mental strategies.
- For children to select and utilise methods of written calculation and manipulatives independently and be given the freedom and encouragement to develop their own methods. Although each method will be taught in the year group specified, children should not be discouraged from using previously taught methods with which they are secure, while the new concepts are becoming embedded. In addition if children are secure in one form of calculation differentiation should provide children the opportunity to progress to a more sophisticated form.
- For children to reflect upon which method to use to solve a problem and ask questions such as 'Can I do this in my head?' 'Can I do this in my head or do I need equipment to help me?', 'Do I need to use a written method?' then 'Is my answer sensible?'
- For children to be able to clearly explain methods of recording/representation and justify why their answers are correct using sound mathematical vocabulary. Therefore strong speaking and listening opportunities underpin good mathematics teaching.
- For KS2 children to develop an efficient, reliable, compact written method of calculation for each operation that they can apply with confidence and understanding when undertaking calculations that they cannot carry out solely mentally.
- To share progress in written calculations with parents so that they have the confidence and knowledge to support their children at home with their mathematical development.

The objectives and content for this policy was originally provided by Chartwell Primary Maths September 2013

Multiplication

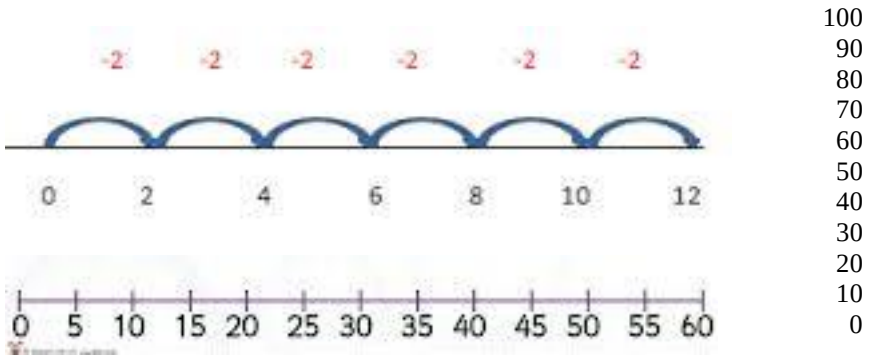
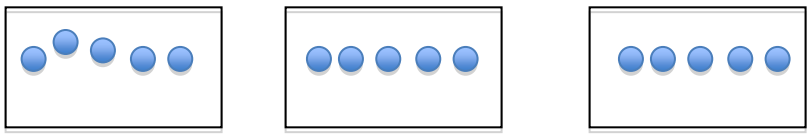
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication, which they know they can rely on when mental methods are not appropriate.

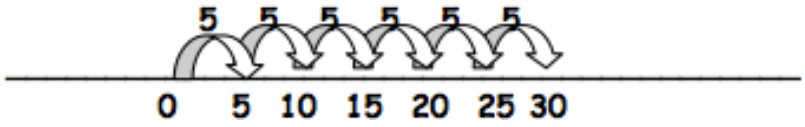
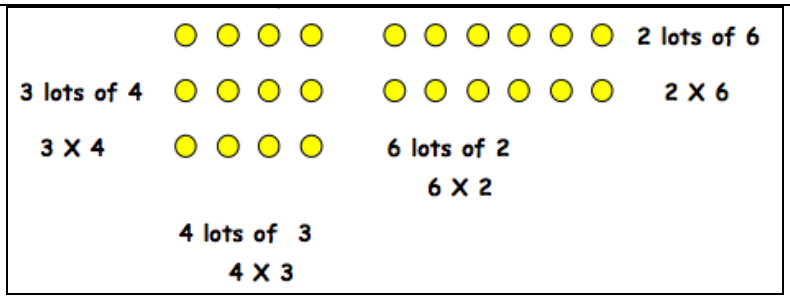
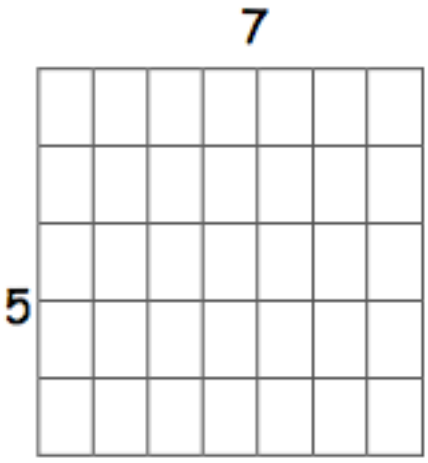
To multiply successfully, children need to be able to:

- Recall all multiplication facts to 12x12;
- Partition numbers into multiples of one hundred, ten and one;
- Apply multiplication facts e.g. 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- Double numbers to 10, 20, 50, 100, 500 progressively
- Multiply by 10,100, 1000 by shifting digits 1, 2, 3, place value to the left respectively
- Utilise closely related multiplication facts e.g. 13 x 11 = (13 x 10) + 13 = 130 +13 = 143 and 40 x 30 = (4x3) x 100 = 1200 and later with decimals 4x0.3 = (4x3)÷10 = 1.2
- Add two or more single-digit numbers mentally;
- Add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- Add combinations of whole numbers using the column method (see above).
- Square numbers and prime numbers

It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for multiplication.

Developing the mental image of multiplication

Putting objects into equal groups Putting objects into equal groups and then checking there are for example, 2 in each group. Begin counting in equal steps by counting the number in 2 groups and then 3 and then 4 etc…	 2 4 6 8
Counting in equal steps, starting with 2s, 10s and 5s, then progressing to 3s, 4s and then 6s, 7s, 8s and 9s Using practical apparatus such as Numicon.	 3 6 9 12
Counting in equal steps, starting with 2s, 10s and 5s, then progressing to 3s, 4s and then 6s, 7s, 8s and 9s Understanding how to count in these steps is an important foundation to learning multiplication facts (tables)	 -2 -2 -2 -2 -2 -2 0 2 4 6 8 10 12 0 5 10 15 20 25 30 35 40 45 50 55 60 100 90 80 70 60 50 40 30 20 10 0
Multiplication as repeated addition 5 x 3 = There are 5 cakes in a pack. How many cakes in 3 packs? Dots or tally marks are often drawn in groups. This shows 3 groups of 5.	 5 + 5 + 5

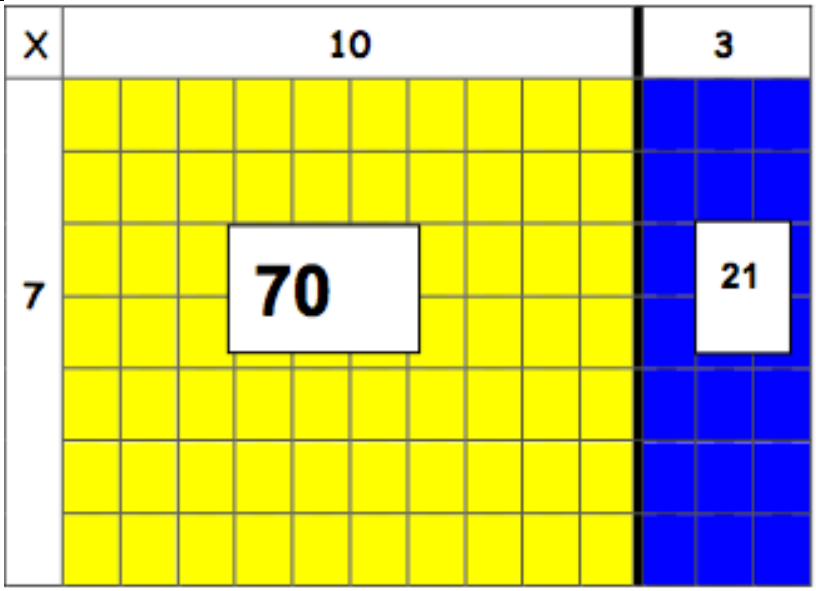
Number lines This model illustrates how multiplication relates to repeated addition Pattern work on a 100 square helps children begin to recognise multiples and rules of divisibility Using Numicon number line to solve repeated addition problems by laying pieces upon track.	6 x 5 = 30 or 5 + 5 + 5 + 5 + 5 + 5 = 30 
Arrays Successful written methods depend on visualising multiplication as a rectangular array. It also helps children to understand why 3 x 4 = 4 x 3	
Rectangular Arrays The rectangular array gives a good visual model for multiplication. The area can be found by repeated addition (in this case 7+7+7+7+7) Children should then commit 7 x 5 to memory and know that it is the same as 5 x 7 Area models like this discourage the use of repeated addition. The focus is on the multiplication facts	

Mental multiplication using arrays and partitioning to multiply a two- digit number by a one-digit number

An array illustrates the distributive law of multiplication i.e.

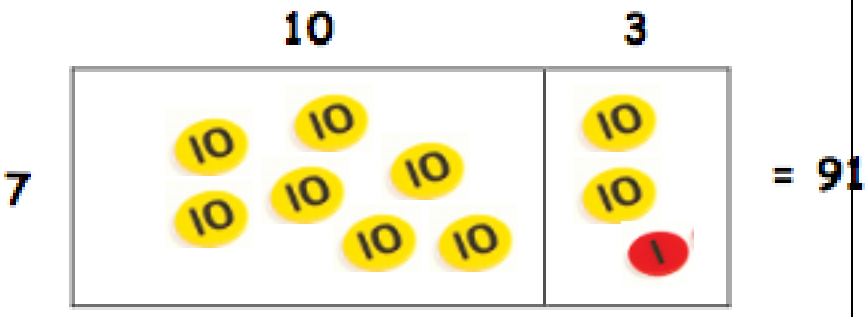
13 x 7 is the same as (10 x 7) + (3 x 7)

Please note that the squares are used to ensure that children have a secure mental image of why the distributive law works



Grid Method using place value counters.

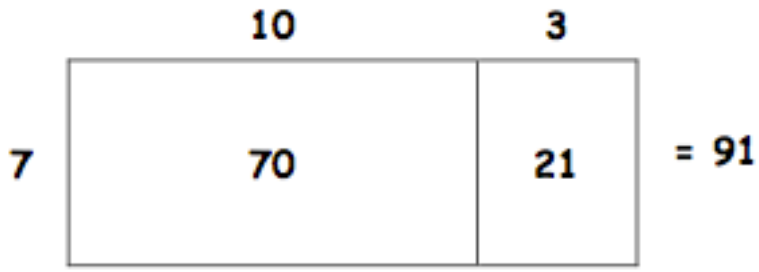
13 x 7 = 91

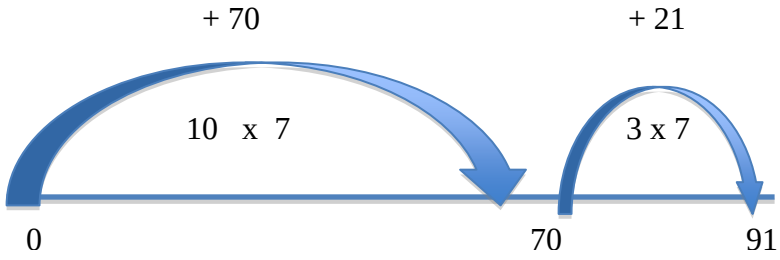


This can lead to the use of a "blank rectangle/open arrays" to illustrate

13 x 7 = (10 x 7) + (3 x 7)

Note the rectangle is drawn to emphasise the comparative size of the numbers



Alternatively a number line can be used	
<u>Using the grid method to multiply two- digit by one-digit numbers</u> At first children will probably need to partition into 10's (example A) It is important, if they are to use a more compact method, that they can multiply multiples of 10 (example B) i.e. 38 x 7 they must be able to calculate 30 x 7 as well as 8 x 7 **Note the grid is drawn to emphasise the comparative size of the numbers**	**38 X 7 is approximately 40 X 7 = 280** Example A 10 10 10 8 7 | 70 | 70 | 70 | 56 Example B 30 8 7 | 210 | 56 =266 Leading to the layout X | 30 | 8 7 | 210 | 56 = 266 **This will lead to a more formalised layout**

Two-digit by two-digit products using the grid method

Extend to TU × TU, asking children to estimate first.

Start by completing the grid. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product

Please note that at this stage the grid is no longer drawn to reflect the respective size of the digits. If a child shows signs of insecurity return to rectangular arrays to ensure understanding

38 x 14

X	30	8	
10	300	80	380
4	120	32	152
			532

Three-digit by two-digit products using the grid method

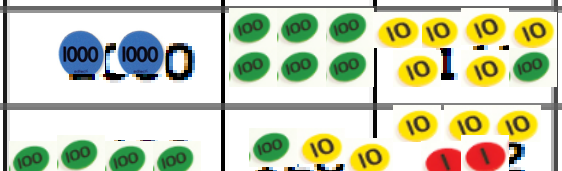
Extend to HTU × TU asking children to estimate first.

Ensure that children can explain why this method works and where the numbers and the grid come from

Place Value counters can help children who are less secure in their number facts.

138 x 24 =

X	100	30	8	
20	2000			2760
4				552
				3312



X	100	30	8	
20	2000	600	160	2760
4	400	120	32	552
				3312

<table>
<tr>
<td>The grid method works just as satisfactorily with decimal numbers as long as the children can apply their knowledge of multiplication facts to decimal numbers.</td>
<td>38.5 x 24 =

<table>
<tr><td>X</td><td>30</td><td>8</td><td>0.5</td><td></td></tr>
<tr><td>20</td><td>600</td><td>160</td><td>10</td><td>770</td></tr>
<tr><td>4</td><td>120</td><td>32</td><td>2</td><td>154</td></tr>
<tr><td></td><td></td><td></td><td></td><td>924</td></tr>
</table>
</td>
</tr>
<tr>
<td colspan="2">It will be down to the class teacher as to whether they move onto the next stage with their pupils. Children need to be confident with the grid method before this can be considered.</td>
</tr>
<tr>
<td><u>Expanded short multiplication leading to column method</u>

The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.

Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.</td>
<td>38 x 7 is approximately 40 x 7 = 280

30 + 8
x 7
210 (30 x 7)
56 (8 x 7)
266

T U
3 8
x 7
210
56
266</td>
</tr>
</table>

<table>
<tr>
<td><u>Short multiplication</u>

The recording is reduced further, with carry digits recorded below the line

If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of the grid method</td>
<td>38 x 7 is approximately 40 x 7 = 280

3 8
<u>x 7</u>
<u>2 6 6</u>
5

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two- digit or three-digit number mentally before they reach this stage.</td>
</tr>
<tr>
<td><u>Multiplying two-digit by two-digit numbers</u> includes the working to emphasise the link to the grid method</td>
<td>56 × 27 is approximately 60 × 30 = 1800

5 6
<u>x 2 7</u>
42 (6 X 7)
350 (50 X 7)
120 (6 X 20)
<u>1000 (50 X 20)</u>
1512
1</td>
</tr>
<tr>
<td><u>Three-digit by two-digit numbers</u>

Continue to show working to link to the grid method.
This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive for more confident children to move on to a more compact method</td>
<td>2 8 6
<u>X 2 9</u>
54 (6 X 9)
720 (80 X 9)
1800 (200 X 9)
120 (6 X 20)
1600 (80 X 20)
<u>4000</u> (200 X 20)
<u>8294</u>
2</td>
</tr>
<tr>
<td colspan="2">If secure with the expanded method, and children are making very few errors, then they can move on to the compact method. This is at the discretion of the class teacher.</td>
</tr>
<tr>
<td><u>Optional: Compact method for TU x TU and HTU x TU</u></td>
<td>2 3
x <u>1 2</u>
4 6 (2 x 23)
<u>2 3 0 (10 x 23)</u>
2 7 6

1 2 3
x <u>1 2</u>
2 4 6 (2 x 123)
<u>1 2 3 0 (10 x 123)</u>
1 4 7 6</td>
</tr>
</table>

Division

The aim is that children use mental methods when appropriate but, for calculations that they cannot do in their heads, they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

To divide successfully in their heads, children need to be able to:

- Understand and use the vocabulary of division
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- Halves of numbers to 20, 50, 100, 200, 500 progressively
- Recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- Use and apply division facts e.g. 60 ÷ 3 = (6 ÷ 3) x 10 = 20
- Know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- Understand and use multiplication and division as inverse operations.
- **U**nderstand division as repeated subtraction (Grouping)
- Know square roots
- Use factors to help divide e.g. 378÷21 = . (378÷3) ÷7 = 18
- Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- Know subtraction facts to 20 and to use this knowledge to subtract multiples of 10 e.g.120 - 80, 320 – 90

Please see Math Assessment Proforma for specific age related mental strategies for children to acquire.

It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for division. To carry out written methods of division successfully, children also need to be able to:

<table>
<tr>
<td><u>Division by sharing</u>

Practical sharing along with more pictures and jottings.

6 strawberries shared between 2 children. How many strawberries do they get each?
6 ÷ 2 =
  </td>
<td> 

I had 10 fish treats shared between 5 fish. 10 ÷ 5 = 2

Two boys shared 20 bananas. How many did they get each?
20 ÷ 2 = 10</td>
</tr>
<tr>
<td colspan="2">Sharing should only be used briefly as a precursor to grouping, which is a more preferable method and should be moved onto as soon as possible. Solving division by grouping strengthens mental calculation strategies.</td>
</tr>
<tr>
<td><u>Division by grouping</u>

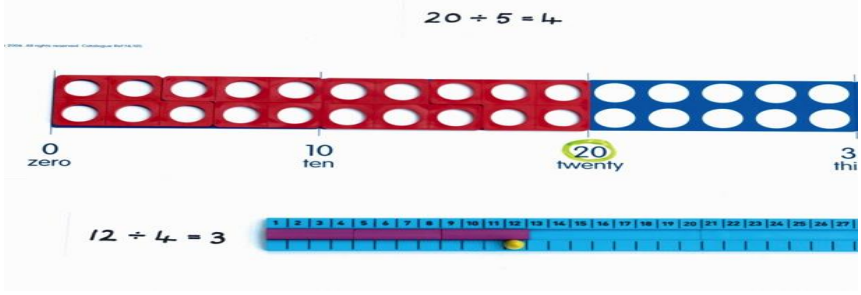
4 apples are packed in a basket. How many baskets can you fill with 12 apples?
12÷4 =</td>
<td>Practical grouping with 12 objects, grouped into 4's. Dots or tally's marks can be split up into groups.
E.g. draw 12 dots representing apples and grouping into 4's to find how many groups. 12÷4 =3

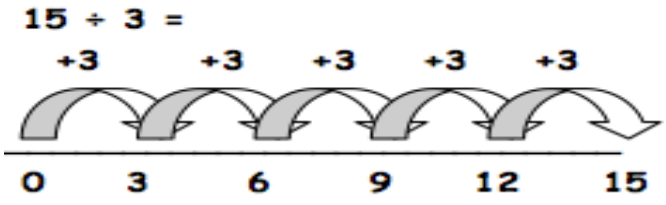
<table>
<tr><td>0000</td><td>0000</td><td>0000</td></tr>
</table>
</td>
</tr>
<tr>
<td><u>Manipulatives number line</u>

How many 5's are in 20?

<u>Number lines (Repeated +)</u>

Counting on in equal steps based on adding multiples up to the number to be divided
Counting back in equal steps based on subtracting multiples from the number to be divided

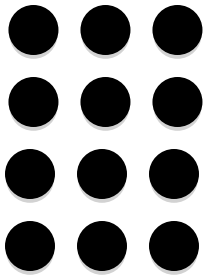
Note Counting on is a powerful tool for mental calculation but does not lead onto written calculation for division</td>
<td> 

A chocolate bar costs 3p. How many can I purchase for 15p?</td>
</tr>
</table>

Using an array to divide

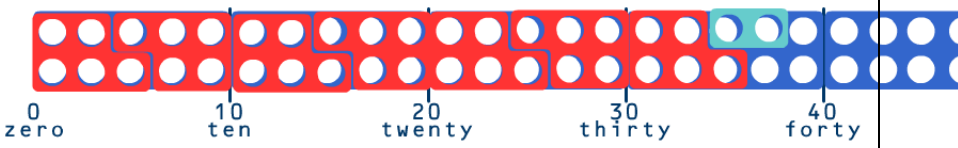
Children can build upon what they have learnt about arrays in multiplication, and it is vital that links between multiplication and division are made in a visual way



12 dots arranged into rows of 3
12÷3=
There are 4 rows/groups so the answer is 4

Finding remainders

37 ÷ 5 = 7 r 2



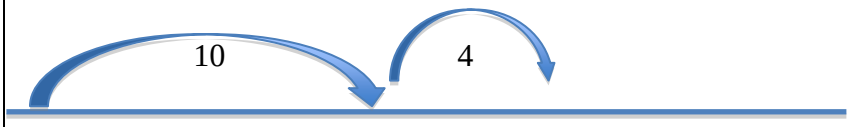
Counting on by chunking

This method is based on adding multiples of the divisor, or 'chunks'. Initially children add several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to add.

Chunking is useful for reminding children of the link between division and repeated addition.

100 ÷ 7 =

10 x 7 = 70 **4 x7 = 28**



0 **70** **98**

Answer 14 remainder 2

As you record the division, ask: 'How many sixes in 100?' as well as 'What is 100 divided by 6?'

Initially children add several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to add.

Children need to recognise that chunking is inefficient if too many additions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples

Answer 33 remainder 2
As you record the division, ask 'How many sixes in 200?' as well as 'What is 200 divided by 6?' Leading to: 200 ÷ 6

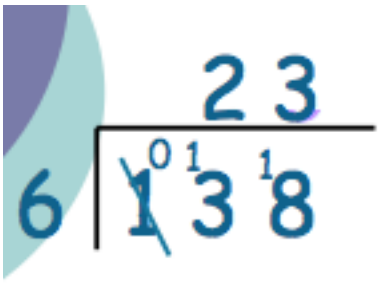
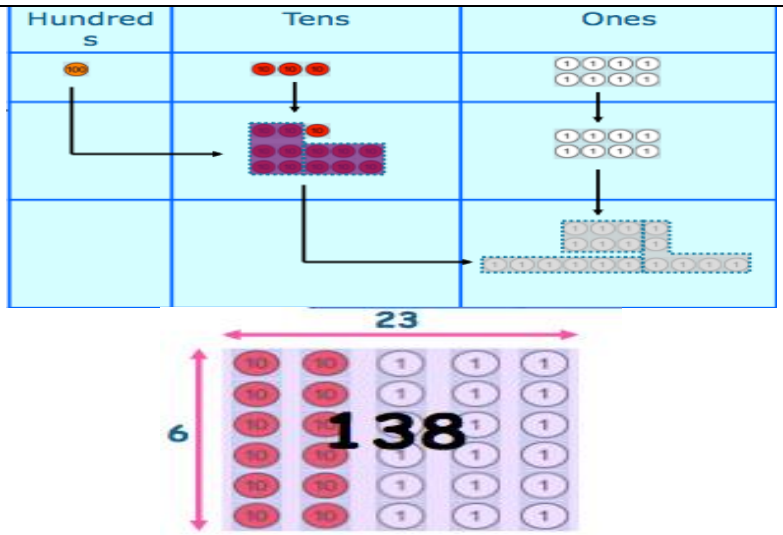
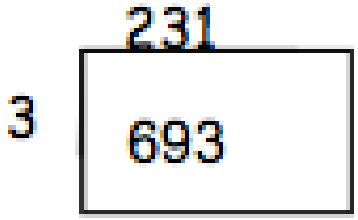
10 x 6 = 60 10 x 6 = 60 10 x 6 = 60 1 x 6 1 x 6 1 x 6

0 60 120 180 186 192 198

30 x 6 = 180 3 x 6 = 18

0 180 198

How many 6's in 200?
200 divided by 6
33 with a remainder of 2

Bus shelter method using place value counters.  	
Bus shelter method for when dividing by a 1-digit number Children should first be introduced to this method by working through calculations where there are no remainders Children should then solve calculations with remainders Children can look at putting remainders into decimals using this method	For example: 693 ÷ 3 =  For example: 937÷ 3 = 312 r 1 3 | 937 For example: 937÷ 3 = 312 .33 (2d.p) 3 | 937 .1 0 1 0
It would be advisable, at this point to return to chunking on a number line with chunking 10 x TU	
Long division The next step is to tackle HTU ÷ TU, which for most children will be in Year 6, where appropriate.	How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. 23 r 8 2 4 | 5 6 0 4 8 0 (20 packs taken) 8 0 7 2 (3 packs taken) 8 (8 remainder)